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CONTRIBUTION OF NON-ISOTHERMAL JETS TO THE PROCESSES OF NOISE GENERATION OF ENERGY MACHINES WHEN INSTALLING SILENCERS

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Abstract: *The method of James Lighthill is known and widely used, which allows determining the acoustic power of isothermal jets. A mathematical model for calculating the acoustic parameters (sound power, radiation pattern) of non-isothermal sound jets is proposed, taking into account the noise silencer installed in the gas exhaust tract. At the output, the equations of continuity, the amount of motion, energy, as well as the Lighthill wave equation are used. A statistical model is used as a turbulence model for calculations. A physical mechanism of noise generation by turbulent flows is proposed, which consists in considering "own" and "shear" noise. The " own " noise is caused by turbulent pulsations of the gas-dynamic flow, the "shift" noise is caused by the presence of a flow velocity gradient. Analytical dependences of the components of "own" and "shift" noise are obtained.*

Keywords: *non-isothermal jet, acoustic power, the continuity equation, equation of the amount of motion, the energy equation, the Lighthill wave equation, "own" noise, "shift" noise*

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1. INTRODUCTION

Noise silencers are a device for reducing noise in gases released into the atmosphere. The principle of operation of such devices is based on a gradual decrease in this pressure or a corresponding decrease in the exhaust gas velocity to a value less than the speed of sound. A muffler is a complex structural element, for the calculation of which it is necessary to take into account the features of its own and shear noise, turbulent features, and so on.

2. CALCULATION OF GAS-DYNAMIC PARAMETERS OF TURBULENT JETS

This section presents a mathematical model for calculating the gas-dynamic parameters of non-isothermal turbulent jets of combustion products and the parameters of acoustic fields generated by these jets. A detailed description of the physical and mathematical model is given in the monograph [1].

The basic equations for describing processes in a turbulent jet are given below [2]:
- the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho V_j) = 0 \tag{1}$$

- equation of the amount of motion:

$$\frac{\partial(\rho V_j)}{\partial t} + \frac{\partial}{\partial x_j}(\rho V_j V_k) = -\frac{\partial p}{\partial x_k} + \frac{\partial}{\partial x_j}\tau_{jk} \tag{2}$$

- the energy equation:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}(\rho V_j h) = \frac{\partial p}{\partial t} + V_k\frac{\partial p}{\partial x_k} + \tau_{jk}\frac{\partial V_k}{\partial x_j} + \frac{\partial q_j}{\partial x_j} \tag{3}$$

where:

$\tau_{jk} = -\delta_{jk}\frac{2}{3}\mu\frac{\partial V_j}{\partial x_j} + \mu(\frac{\partial V_j}{\partial x_k} + \frac{\partial V_k}{\partial x_j})$ – components of the viscous tangential stress tensor;

$\delta_{jk} = \begin{cases} 1 \text{ by } j = k \\ 0 \text{ by } j \neq k \end{cases}$ Kronecker symbol;

where: μ dynamic viscosity coefficient, Pa·s;
$j, k=1,2,3$ indexes that determine the direction of the axes;
V_j, V_k speed components;
p, ρ, t pressure, density, time;
q_j components of the heat flow vector.

According to the Reynolds model, the instantaneous values of any gas-dynamic parameters$(V_j, p, \rho, \tau_{jk}, q_j)$ can be represented as the sum of the time-averaged$(\overline{V}_j, \overline{p}, \overline{\rho}, \overline{\tau}_{jk}, \overline{q}_j)$ and the pulsation component $(V_{pj}, p_p, \rho_p, \tau_{pjk}, q_{pj})$:

$$V_j = \overline{V}_j + V_{pj};$$
$$p = \overline{p} + p_p;$$
$$\rho = \overline{\rho} + \rho_p;$$
$$\tau_{jk} = \overline{\tau}_{jk} + \tau_{pjk};$$
$$q_j = \overline{q}_j + q_{pj},$$

where
($-$) means averaging over time.

We use the assumptions made for the equations 1-3

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_j}(\overline{\rho}\overline{V}_j) + \frac{\partial}{\partial x_j}(\rho_p V_{pj}) = 0 \tag{4}$$

$$\overline{\rho}\frac{\partial \overline{V}_k}{\partial t} + \overline{\rho}\overline{V}_j\frac{\partial \overline{V}_k}{\partial x_j} = 0 \tag{5}$$

$$\overline{\rho}\frac{\partial \overline{h}}{\partial t} + \overline{\rho}\overline{V}_j\frac{\partial \overline{h}}{\partial x_j} = \frac{\partial \overline{p}}{\partial t} + \frac{\partial \overline{q}_j}{\partial x_j} + \overline{V}_k\frac{\partial \overline{p}}{\partial x_k} + \overline{V}_{pk}\frac{\partial \overline{p_p}}{\partial x_k} + \overline{\tau}_{jk}\frac{\partial \overline{V}_k}{\partial t} \tag{6}$$

$$+ \overline{\tau}_{pjk}\frac{\partial \overline{V}_{pk}}{\partial t} - \frac{\partial}{\partial t}\overline{\rho_p h_p} - \frac{\partial}{\partial x_j}\overline{\rho_p V_{jp} h_p} - \overline{\rho_p V_{pj}}\frac{\partial h}{\partial x_j}$$

$$\overline{p} = \overline{\rho}\overline{R}T \tag{7}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\overline{\rho V_{pi} V_{pk}} + \frac{\partial}{\partial x_j}\left(\overline{V_j \rho V_{pi} V_{pk}}\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho V_{pi} V_{pj} V_{pk}}\right) = -\overline{V_{pj}\frac{\partial p_p}{\partial x_k}} - \overline{V_{pk}\frac{\partial p_p}{\partial x_i}} \\ &+ \overline{V_{pi}\frac{\partial \tau_{pjk}}{\partial x_j}} + \overline{V_{pk}\frac{\partial \tau_{pij}}{\partial x_j}} + \overline{V}_i\frac{\partial}{\partial x_j}\left(V_j \rho_p V_{pk}\right) + \overline{V_k}\frac{\partial}{\partial x_j}\overline{\left(V_j \rho_p V_{pi}\right)} + \overline{V_i}\frac{\partial}{\partial t}\overline{\rho_p V_{pk}} \\ &+ \overline{V_k}\frac{\partial}{\partial t}\overline{\rho_p V_{pi}} + \overline{\rho_p V_{pj}}\frac{\partial}{\partial x_j}\overline{V_i V_k} - \overline{\rho V_{pj} V_{pk}}\frac{\partial \overline{V_i}}{\partial x_j} - \overline{\rho V_{pi} V_{pj}}\frac{\partial \overline{V_k}}{\partial x_j}\end{aligned} \tag{8}$$

A statistical model is used as a turbulence model for calculating gas-dynamic parameters. The physical model is formulated as follows. In the initial section, the flow under consideration is a set of point formations – quasiparticles. The movement of such a particle downstream is random, while the particle retains all its individual properties. To describe the probabilistic trajectory of a particle, it is generally necessary to set a multidimensional probability density, which is the joint probability density of a particle hitting from point A of the initial section sequentially to random points B, C, D, etc.

Without going into details, we can say that the above system of equations allows us to determine all the gas-dynamic parameters and the scale of turbulence in the jet flow field.

Lighthill proposed approximate formulas for calculating the acoustic power emitted by a gas stream, having the form [3-6]:

$$N_{ac} = k\frac{\rho_a^2 V_a^8 d_a^2}{\rho_0 a_0^5}, by\ V_a \geq 150\frac{m}{s} \tag{9}$$

$$N_{ac} = k\frac{\rho_a^2 V_a^6 d_a^2}{\rho_0 a_0^3}, by\ V_a \geq 150\frac{m}{s} \tag{10}$$

where:
K=(2,5÷4,5)•10^-5 – dimensionless coefficient;
ρ_a, V_a – density and speed on the exhaust pipe cut;
ρ_0, a_0 – density and speed of sound in an undisturbed medium;
d_a – diameter of the output section [m].

To calculate the vibroacoustic parameters of non-isothermal jets, we supplement the system of equations (9-10) with the wave equation (the Lighthill wave equation):

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2\frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2}{\partial x_i \partial x_j}\left[\rho V_i V_j - \delta_{ij}(\rho a_0^2 + p) - \mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\frac{\partial V_k}{\partial x_k}\delta_{ij}\right)\right] \tag{11}$$

where
ρ, p – density, static pressure,
a_0 – the speed of sound in an undisturbed environment,
V – speed,
t – time,
x_i, x_j – coordinates, ***i, j, k, l = 1,2,3***.

If the right part of equation (11) is known to us, i.e. the distribution of gas-dynamic parameters in the flow is known, the solution for the energy ***N(θ,Ø)*** radiated by the flow per unit time to a point with coordinates ***x, θ, Ø***, is constructed by classical acoustics methods in the following form [7]:

$$N(\theta, \phi) = \int N(\theta, \phi, y) d^3 y$$
$$N(\theta, \phi, x) = \frac{x_i x_j x_k' x_l'}{16\pi^2 \rho_0 a_0^5 |x|^4}\int \frac{\partial^4}{\partial \tau^4}\overline{T_{ij} T_{kl}'}\, d^3 y \tag{12}$$

where:
N – radiated acoustic power;

$$T_{ij} = \rho V_i V_j - \delta_{ij}(\rho a_0^2 + p) - \mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\frac{\partial V_k}{\partial x_k}\delta_{ij}\right) \tag{13}$$

T_{ij}, T'_{kl} – voltage tensors related to various sound sources in the flow (13)

After simple transformations, taking into account the decomposition of gas-dynamic parameters into the average and pulsation components of noise and neglecting the viscous stress tensor in (13), the integrand in (12) has the form [1]:

$$\begin{aligned}\overline{T_{ij} T_{kl}'} &= \overline{[\overline{\rho} V_i V_j - \delta_{ij}(\rho a_0^2 - p)][\rho V_k' V_l' - \delta_{kl}(\rho' a_0^2 - p')]} = \overline{\rho\rho' V_i V_j V_k' V_l'} \\ &- \delta_{ij} a_0^2 \overline{\rho\rho' V_k' V_l'} - \delta_{kl} a_0^2 \overline{\rho\rho' V_i V_j} + \delta_{kl}\overline{\rho\rho' V_i V_j} + \delta_{ij}\overline{\rho\rho' V_k' V_l'} \\ &+ \delta_{ijkl}\left(a_0^4 \overline{\rho\rho'} - a_0^2 \overline{\rho\rho'} - a_0^2 \rho' p + pp'\right)\end{aligned} \tag{14}$$

3. SPATIAL-TEMPORAL CORRELATIONS OF PULSATION PARAMETERS

In order to fully describe the radiation sources, it is necessary to have information about the mutual space-time correlations and autocorrelations of the average and pulsation components of gas dynamic quantities.

To date, the question of the types and forms of correlation relationships in turbulent flows has not been fully studied. This method is based on the data of [2, 8-10], taking into account the uniformity, isotropy and compressibility of the flow. All space-time correlations of the pulsation parameters can be represented as:

$$\overline{S_\iota S_k'}(x_p, z, \tau) = f(x_p, z) g(x_p, \tau) \sqrt{\overline{S_\iota^2}} \sqrt{\overline{S_k^2}} \qquad (15)$$

where:

$f = e^{-\frac{\pi z^2}{L^2}}$	spatial factor,
$g = e^{-\omega\tau}$	time factor,
$r^2 = r_1^2 + r_2^2 + r_3^2$-	the distance between the correlated points,
$\omega = \frac{1}{T_E}$	characteristic frequency of turbulent pulsations,
L, T_E	typical turbulence scales.

Tab. 1 shows the analytical dependences obtained taking into account the space-time correlation between the gas-dynamic parameters for 9 summand included in the product of tensors (15), reflecting the contribution of various gas-dynamic parameters (pulsation, average), each elementary volume of the jet to acoustic radiation.

Tab. 1 uses the terminology of the physical mechanisms of noise generation by turbulent flows accepted in modern aero-acoustics: "own" and "shear" noise. The "own " noise is caused by turbulent pulsations of gas-dynamic parameters. The "shift" noise is caused by the presence of a flow velocity gradient [1].

The presented mathematical model allows performing calculations of acoustic parameters (sound pressure, acoustic power, intensity, radiation pattern, etc.) generated by gas flows.

		Own noise	Shift noise
1	$\overline{\rho\rho' V_\iota' V_J' V_k' V_l'}$	$\frac{1}{2}\omega^4 L^3 V_P^4 [(-23\cos^4\theta + (30 + 72\sqrt{3})\cos^2\theta + 11 + 36\sqrt{3})\rho_p^2 + 8\sqrt{2}]$	$\omega^4 L^3 \tilde{V} \cos^2\theta (4\tilde{V}\tilde{\rho} V_p^2 \rho_p \cos^2\theta + 4(3 + 8\sqrt{2})\tilde{\rho} V_p^3 \rho_p + \tilde{V}^3 \rho_p^2 \cos^2\theta + 4\tilde{V}\tilde{\rho}^{2\cdot} V_p^2 + 2\tilde{V} V_p^2 \rho^2 \times [(2 + + 8\sqrt{2})\cos^2\theta + 1 + 8\sqrt{2}])$
2	$\overline{\delta_{kl}^* V_\iota V_J \rho\rho'}$	$\omega^4 L^3 V_p^2 \rho_p p_p$	$\omega^4 L^3 \tilde{V} p_p (\tilde{V}\rho_p + 2V_p\tilde{\rho})\cos^2\theta \times (2 - \cos^2\theta)$
3	$\overline{\delta_{\iota J}^* V_k' V_l' \rho' \rho}$	$\omega^4 L^3 V_p^2 \rho_p p_p \times (2\cos^4\theta + 1)$	$\omega^4 L^3 \tilde{V} p_p (\tilde{V}\rho_p + 2V_p\tilde{\rho})\cos^4\theta$
4	$\overline{\delta_{\iota J}^* \delta_{kl}^* p p'}$	$\omega^4 L^3 p_p^2$	–
5	$\overline{-a_0^2 \delta_{kl}^* \rho V_\iota V_J \rho'}$	$-\omega^4 L^3 a_0^2 V_p^2 (2\cos^4\theta - 4\cos^2\theta - 1)\rho_p^{2\prime}$	$-\omega^4 L^3 a_0^2 \tilde{V} \rho_p (\tilde{V}\rho_p + 2V_p\tilde{\rho})\cos^2\theta \times (2 - \cos^2\theta)$
6	$\overline{-a_0^2 \delta_{\iota J}^* \rho V_k' V_l' \rho'}$	$\omega^4 L^3 a_0^2 V_p^2 \rho_p^2 (2\cos^4\theta + 1)$	$-\omega^4 L^3 a_0^2 \tilde{V} \rho_p (\tilde{V}\rho_p + 2V_p\tilde{\rho})\cos^2\theta$
7	$\overline{a_0^4 \delta_{\iota J}^* \delta_{kl}^* \rho\rho'}$	$\omega^4 L^3 a_0^4 \rho_p^2$	–
8	$\overline{-a_0^2 \delta_{\iota J}^* \delta_{kl}^* \rho p'}$	$-\omega^4 L^3 a_0^2 \rho_p p_p$	–
9	$\overline{-a_0^2 \delta_{\iota J}^* \delta_{kl}^* \rho' p}$	$-\omega^4 L^3 a_0^2 \rho_p p_p$	–

Tab. 1: Analytical dependences of the components of "own" and "shift" noise

4. CONCLUSION

A mathematical model for calculating the acoustic parameters of non-isothermal jets flowing from a silencer is developed. The equations of continuity, the amount of motion, energy, as well as the wave equation of James Lighthill are used. Analytical expressions for determining the "shift" noise and "intrinsic" noise are obtained.

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