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THEORETICAL STUDY OF THE CLOSED BAR SYSTEM LOSS FACTOR

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Abstract: *The coefficient of vibrational energy loss is a physical and mechanical characteristic, the accuracy of calculation of vibroacoustic characteristics of corresponding machines at the stage of their design to a great extent depends on its value.*

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1. INTRODUCTION

Currently, this value is studied for U-sections, angles and plates [1-11]. These data were obtained for some of the above-mentioned elements on a special stand. It should be mentioned that such studies have not been carried out for energetically closed systems (in this case, bars). Therefore, the purpose of studies, the results of which are given in this article, was to clarify the dissipative function of the bar system, which is a closed energy system itself.

2. EQUIPMENT AND DEVICES USED IN THE RESEARCH

The internal friction inherent in real materials causes the loss of energy in the current cycles of vibration of the system. To account for the total dissipation of energy, leading to a decrease in the specific energy per unit time, we introduce its evaluation, taking into account both internal and structural energy losses of the considered acoustic systems. A constant value called the loss factor (***η***) is taken as a quantitative characteristic of this estimation. The loss coefficient and the energy attenuation coefficient (***δ***) considered earlier are interrelated in harmonic motion as follows:

$$\boldsymbol{\delta = \frac{1}{2}\omega\eta = \frac{2\pi f\eta}{2} = \pi f\eta} \tag{1}$$

where
f is the vibration frequency.

The loss coefficient shows what fraction of the total energy inherent in the element is lost during one complete period of oscillation and finds a relationship with the energy parameters by means of the ratios:

$$\begin{aligned}\mathbf{\frac{dE}{dt} = \pi E\eta}\\ \mathbf{\frac{dU}{dt} = \pi U\eta}\end{aligned} \tag{2}$$

where
E and ***U***, respectively, are the total and potential energies of the element included in the system.

For the occurrence of flow under these conditions, the presence of an external force that would remove the bar from the state of equilibrium is necessary. A mere change in the internal energy state of the coupled bars due to losses will not lead to the emergence of such a force. Consequently, the elements of the bar system will oscillate in the mode corresponding to itself in the presence of internal losses. The time of this damping due to losses, during which the amplitude of oscillations will decrease "e" times, can be determined from the expression:

$$\mathbf{t}_d = \mathbf{\frac{1}{\pi f\eta}} \tag{3}$$

From where it is clear that the damping time depends on the frequency and the loss factor. This time, on the one hand, should be much longer than the time required for the energy flows through the adjacent boundary of the coupled rods to go to zero. At the same time, on the other hand, it should be longer, at least by an order of magnitude of time($\mathbf{t} = \frac{10\,\mathbf{L}_{tot}}{\mathbf{c}}$, where L_{tot} is the total length of the system), necessary for the multiple origin of the elastic wave along the entire length of the system. Otherwise, the system we are considering will not satisfy the isolation conditions due to sufficiently large energy losses.

Further note that for each of the rods included in the system, we can write down the step-by-step (cycle-by-cycle) reduction of energy due to its internal losses

$$\mathbf{U\eta}, \mathbf{U\eta^2}, \mathbf{U\eta^3}, \dots, \mathbf{U\eta^n}$$

where

U is the value of potential energy of the element under consideration in the system;

n is the number of full cycles of passage by the elastic wave at the time of observation.

3. RESULTS AND DISCUSSION

The above sequence can be used to estimate the amount of absorbed (***U***) (loss) energy for a finite period of time, for which we make an expression:

$$\mathbf{U}_{le} = \mathbf{U\eta} + \mathbf{U\eta^2} + \dots + \mathbf{U\eta^n} = \mathbf{U\eta}\frac{(\mathbf{1}-\mathbf{\eta^n})}{\mathbf{1}-\mathbf{\eta}} \tag{4}$$

Where the right (last) part is the result of summing the finite segment of a geometric progression series with denominator η less than unity. The limiting period at aspiration gives a finite amount of transverse energy, defined by the expression

$$\mathbf{U}_{le} = \mathbf{U}\frac{\mathbf{\eta}}{\mathbf{1}-\mathbf{\eta}} \tag{5}$$

Taking into account that the loss factor η for most engineering materials is, as a rule, much less than unity, the last expression will be written in the form

$$\mathbf{U_{le}} = \mathbf{U\eta}\frac{\mathbf{1}}{\mathbf{1}-\mathbf{\eta}} = \mathbf{U\eta}(\mathbf{1}-\mathbf{\eta})^{-1} = \mathbf{U}(\mathbf{\eta}+\mathbf{\eta^2}+\mathbf{\eta^3}+\cdots) \tag{6}$$

With sufficient estimation for practice, let us hold here only the decomposition term, the result being:

$$\mathbf{U}_{le} = \mathbf{U\eta} \tag{7}$$

Analyzing the above relations, it is easy to see that the greatest amount of energy is lost during the first cycle of oscillations, which is also clear from the last equality, which is the result of the limiting operation, and this fact takes place for each element included in the system. At the same time, from the last relations we establish that the total energy of losses (resulting from the limiting operation) tends to its constant value.

The total energy of system losses will be added up from the losses of its individual elements, so for each first current cycle of elastic wave propagation through the system elements (its double stroke), we can write down the energy ratio taking into account the losses of the first main current cycle, i.e.

$$\mathbf{A\eta} = \mathbf{U_1\eta_1} + \mathbf{U_2\eta_2} + \dots + \mathbf{U_n\eta_n} = \sum_{\mathbf{n}} \mathbf{U_n\eta_n} \tag{8}$$

From where, the total loss factor of the system will be represented by the formula

$$\mathbf{\eta}_{tot} = \frac{\mathbf{U_1\eta_1}}{\mathbf{A}} + \frac{\mathbf{U_2\eta_2}}{\mathbf{A}} + \dots + \frac{\mathbf{U_n\eta_n}}{\mathbf{A}} = \frac{\sum_{\mathbf{n}} \mathbf{U_n\eta_n}}{\mathbf{A}} \tag{9}$$

If the loss factors of the individual elements that make up the system are equal to each other, then the total loss factor of the entire system will correspond to the loss factor of any element. If, however, the loss factor on the elements $\mathbf{\eta}_i$ $(\mathbf{i} = \overline{\mathbf{1}, \mathbf{n}})$ are not equal to each other, then the system loss factor ηcomm will be in the range

$$\mathbf{\eta_j} < \mathbf{\eta}_{tot} < \mathbf{\eta_i}$$

where

$\eta_j < \eta_i$

All the above, of course, carries over to a system with

$$\mathbf{U_1} = \mathbf{U_2} = \mathbf{U_3} = \cdots \mathbf{U_n}$$

If the potential energies are not equal to each other, then the loss factor of the system will depend on the condition which losses, by their values, belong to the corresponding energy element, for example, if the equation takes place:

$$\mathbf{U_1} > \mathbf{U_2} > \mathbf{U_3}$$
$$\mathbf{\eta_1} < \mathbf{\eta_2} < \mathbf{\eta_3}$$

Then the total loss coefficient will be close in value to the highest loss coefficient.

Provided that $\mathbf{U_1} > \mathbf{U_2} > \mathbf{U_3}$
but $\mathbf{\eta_1} < \mathbf{\eta_2} < \mathbf{\eta_3}$

Then the total loss factor will lie between the boundaries

$$\mathbf{\eta_1} < \mathbf{\eta_{tot}} < \mathbf{\eta_3} \tag{10}$$

However, its value will be closer to the lowest number.

To summarize, it should be mentioned that in a mechanical system, it is advisable to first impose conditions on the change of losses in the system elements having the greatest potential energy.

At the same time, the limiting total estimation of the value of energy losses in the mechanical system does not influence the character of energy distribution within the system and at the same time, as it is established in work, the presence of losses accelerates the process of approach in system of an energy stationary state at which energy exchange between its elements stops.

It is known that the loss factor, the inverse of the Q-value system, corresponds to the tangent of the phase angle (***φ***). For most materials used in engineering, the angle tangent can be replaced by the value of the total angle due to the smallness of its values, so that

$$\mathbf{\eta} = \mathbf{tg\varphi} \approx \mathbf{\varphi}$$

Since losses are caused by internal friction in the suppression elements of which the material is made, this makes it possible to consider them dependent on each other, at the same time, the stiffness factor (the value inverse of the suppression) is proportional to the elastic modulus, and therefore the elastic and dissipative properties of the suppression element can be characterized by the elasticity complex, which allows to write the equality

$$\mathbf{E^* = \bar{E} = E_0 + iE_0' \approx E_0(1 + i\eta)} \quad (11)$$

This equality is true to within an order of by virtue of the expansion, and since ***η«1***, the values $\boldsymbol{E_0}$ and $|\bar{E}| = E_0\sqrt{1+\eta^2} = E$, should be considered equal between themselves, so the zero index of ***E*** can be omitted to account for the real part.

In accordance with this complex modulus of elasticity let us write it in the form:

$$\mathbf{E^* = E(\alpha + i\beta) = E\left(\frac{1 + itg\varphi}{1 - itg\varphi}\right)} \quad (12)$$

where
φ is the phase angle when losses are taken into account.

By multiplying the numerator and denominator by the conjugate significant expression (***1+itg φ***) we successively obtain:

$$\mathbf{E(\alpha + i\beta) = E\left(\frac{1 + itg\varphi}{1 - itg\varphi}\cdot\frac{1 + itg\varphi}{1 + itg\varphi}\right) = E\frac{(1 + itg\varphi)^2}{1 + tg^2\varphi} = E\left(\frac{1 - tg^2\varphi}{1 + tg^2\varphi} + i\frac{2tg\varphi}{1 + tg^2\varphi}\right)} \quad (13)$$

Then for equality of complex quantities we must have:

$$\mathbf{(^*)\alpha = \frac{1 - tg^2\varphi}{1 + tg^2\varphi} = cos24}$$

$$\mathbf{\beta = \frac{2tg\varphi}{1 + tg^2\varphi} = sin24}$$

$$\mathbf{tg24 = \frac{2tg\varphi}{1 - tg^2\varphi} = \frac{\beta}{\alpha}}$$

$$\mathbf{24 = arctg\frac{\beta}{\alpha}}$$

Given that the tangent of the phase angle corresponds to the loss factor, we can write:

$$\mathbf{\alpha = \frac{1 - \varphi^2}{1 + \varphi^2}}$$

$$\mathbf{\beta = \frac{24}{1 + \varphi^2}}$$

and ***24 = arctg η = η***,
where : ***φ =η/2***

From this we obtain that ***φ*** is the coefficient of internal friction per half-period, corresponding to half of the loss coefficient per cycle.

Between α and β the conditions are fulfilled: modulus $|\mathbf{E^*}| = \mathbf{1}$; argument $\mathbf{E^*(arctE^* = 24)}$.

The transformations give:

$$\mathbf{|E^*| = \sqrt{\alpha^2 + \beta^2} = 1, \ \ arctE = arctg\frac{\beta}{\alpha} = 24 = \eta}$$

The limiting case, according to equality (*), we have when $\mathbf{tg\varphi = 1}$ or $\boldsymbol{\varphi = \frac{\pi}{4}}$, at which ***α*** and ***β*** obtain values ***α=0, β=1***, corresponding to a perfectly plastic body. This result finds confirmation with the studies given in this paper, connected with the energy distribution in the presence of damping and without it, where the equality took place: ***η = arctg (1)***, by the example of considering the interaction of two identical rods. This condition was represented graphically as a sectorless line on the phase plane. Taking equality ***α+β=1*** as the basis, and also taking into account that by analogy with the complex modulus of elasticity the complex velocity of propagation ***c**** is connected with the loss factor by the following relation ***c* = c(1-i η/2)***, which gives the grounds for taking the energy conservation law as the second equality

$$\mathbf{\bar{Q} + \bar{R} = 1; \ \ (\bar{Q}, \bar{R} > 0)}$$

where
$\mathbf{\bar{Q}}$ and
$\mathbf{\bar{R}}$ are energy transition coefficients.

From these two equalities, one, and, putting additionally that $\mathbf{\bar{Q} = a^2; \bar{R} = b^2}$, we show that with these related parameters, the formula

$$\mathbf{f(Q, R, \eta) = |Q\alpha - R\beta|}$$

does not exceed unity element, i.e.$|\mathbf{Q\alpha - R\beta}| \le \mathbf{1}$, accounting for losses.

Indeed, multiplying the above equations term by term, we obtain

$$\mathbf{a^2\alpha^2 - a^2\beta^2 + b^2\alpha^2 + b^2\beta^2 = 1}$$

or

$$\mathbf{a^2\alpha^2 - a^2\beta^2 + b^2\alpha^2 + b^2\beta^2 + 2ab\alpha\beta = 1}$$

Whence $\mathbf{(a\alpha - b\beta)^2 + (b\alpha + a\beta)^2 = 1}$.

Since ***(bα-aβ right)² ≥0***, we have ***(aα-bβ right)² ≤1*** , hence we conclude that

$$\mathbf{\left|\sqrt{\bar{Q}}\cdot\alpha - \sqrt{\bar{R}}\cdot\beta\right| \le 1}$$

considering $\mathbf{\sqrt{\bar{Q}} = Q}$ and $\mathbf{\sqrt{\bar{R}} = R}$.

The obtained last equation establishes the relationship between the transition coefficients ***Q*** and ***R*** and the loss factor η.

It follows that taking into account the loss factor from the point of view of estimating energy absorption accelerates the process of establishing the stationary energy state by the system. Indeed, as shown earlier, the formula

$$\mathbf{W_0|Q - R|^n = f(n)}$$

represents the rate of change in the energy flux for each element of the system as a function of ***n = τ/T*** (the number of cycles at the moment of observation) in cases where energy losses were neglected.

When losses are taken into account for the values of Q and R under consideration, the analogue of the latter expression is the expression $\mathbf{W_0|Q\alpha - R\beta|^n}$, corresponding to the loss-aware flow for the current values of ***n***, which can only decrease as n passes. Let us show this by using the inequality:

$$\mathbf{f(Q, R, \eta) = |Q\alpha - R\beta| \leq |Q - R|}$$

Let us estimate the values of ***α*** and ***β***, whereby we present them as follows, given that ***η«1***

$$\frac{\mathbf{1-\eta^2}}{\mathbf{1+\eta^2}} = \frac{\mathbf{1+\eta^2-2\eta^2}}{\mathbf{1+\eta^2}} = \mathbf{1} - \frac{\mathbf{2\eta^2}}{\mathbf{1+\eta^2}} = \mathbf{1-2\eta^2(1+\eta^2)^{-1} \approx 1-2\eta^2}$$

Here in the expansion of the expression $\frac{\mathbf{1}}{\mathbf{1+\eta^2}} = \mathbf{(1+\eta^2)^{-1}}$, the first two terms are taken into account due to the smallness of ***η***.

$$\mathbf{\beta} = \frac{\mathbf{2\eta}}{\mathbf{1+\eta^2}} = \frac{\mathbf{2}}{\mathbf{\eta + \frac{1}{\eta}}}$$

where we have ***β<1 since η+1/η>2 (η≠1)***.

Thus, we find that ***α*** and ***β*** for any values of the loss factor ***η<1*** are in the interval

$$\mathbf{0 < \alpha < 1}$$

and

$$\mathbf{0 < \beta < 1}$$

And hence we conclude that $\mathbf{|Q\alpha - R\beta| \leq |Q - R|}$,

wherefore

$$\mathbf{W_0|Q\alpha - R\beta|^n \leq W_0|Q - R|^n} \tag{14}$$

4. CONCLUSIONS

Physically, the last inequality means that for all possible cases of interaction of bars included in bar systems, the stationary energy equilibrium state for the system considering the internal energy losses will come earlier than without considering the latter, as it follows from the analysis of the energy direct image problems considered in the paper.

It was also presented that as a result of carried out an analysis of the bar elements system, each of them acquired a certain amount of energy in the time required for this purpose.

With further increase of time, as the results suggest the elements stop exchanging energy, which corresponds to the onset of the moment of stable oscillations of the system, corresponding to the eigenforms.

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