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VIBRATIONS OF THE CLOSED FRAME STRUCTURES IN A STEADY-STATE CONDITION

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***Abstract**: The load-bearing frames of the technological machinery of various functional purposes, such as bridge and gantry cranes, locomotives, motor locomotives, etc., are energetically closed rod systems [1-10].*

***Keywords:** Overhead cranes, noise, vibration, rod systems, energy distribution.*

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1. INTRODUCTION

In the steady-state mode, the energy distribution in the rod system is carried out in accordance with the potential energy of each of the system elements. In other words, there are no energy flows through the boundaries of the elements in the system.

2. EQUIPMENT AND DEVICES USED IN THE RESEARCH

The boundaries of the elements in the system do not perform any work and, therefore, mechanical vibrations can be determined. Independent components that determine the potential energy distribution among the elements of the system can be reduced by some orthogonal transformation to the main (normal) axes. There is an orthogonal basis in $\{\overline{e_1}, \overline{e_2}, \dots, \overline{e_n}\}$, which $U^{i,II}$ is written as:

$$U^{i,II}(\overline{x}) = \frac{1}{2}\sum_{K=1}^{n} a_K^{I,II}\, x_K^2 \tag{1}$$

where

x is a vector of dimension n and when decomposed by basis vectors:

$$\overline{x} = x_1\overline{e_1} + x_2\overline{e_2} + \cdots + x_n\overline{e_n}$$

For each of the system rods.

Here

$x_1, x_2 \dots x_n$ are the coordinates of the vector x in the selected basis.

Thus, for example, when working with a rod system that corresponds to a differential equation.

$$\ddot{\overline{x}}^{(I,II)} = -\overline{grad}\, U^{I,II} \tag{2}$$

In the selected coordinate system, the latter equations will take the form:

$$\begin{aligned} &\ddot{x}_1^I = -a_1^I x_1^I; \quad \ddot{x}_2^I = -a_2^I x_2^I; \quad \dots \quad \ddot{x}_n^I = -a_n^I x_n^I; \\ &\ddot{x}_1^{II} = -a_1^{II} x_1^{II}; \quad \ddot{x}_2^{II} = -a_2^{II} x_2^{II}; \quad \dots \quad \ddot{x}_n^{II} = -a_n^{II} x_n^{II} \end{aligned} \tag{3}$$

3. RESULTS AND DISCUSSION

Thus, it is established that for longitudinal oscillations, the system splits into a number of separate invariant subspaces, which are independent solutions with eigenvalues ***ai(I,II)***, describing eigen-oscillations in each of them.

At the same time, if each of the forms $U^{(I,II)}(x)$ that determine the potential energies of the rods is positive definite, then all $a_k^{(I,II)}$ are positive. Then the considered point x (vector X), in each of the corresponding invariant subspaces, performs n independent oscillations along n mutually perpendicular directions in the corresponding bases selected in them: $e_1^{(I,II)}; e_2^{(I,II)}; \dots, e_n^{(I,II)}$. These oscillations are called the main or eigen-oscillations, and the numbers $\omega_k^{(I,II)}$ are the eigen-frequencies of vibrations for each individual element.

The eigen-frequencies can be found based on the equation of motion of a rod conjugated to another rod:

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = -ku \qquad or \qquad c^2\frac{\partial^2 u}{\partial x^2} - ku = \frac{\partial^2 u}{\partial t^2}. \tag{4}$$

We are looking for a solution in the form of a product of two functions, one of which depends only on x, and the other depends only on t:

$$U(x,t) = X(x)T(t) \tag{4a}$$

Substituting (4a) into (4) gives:

$$c^2T(t)\cdot x^{II}(x)-k\cdot X(x)T(t)=X(x)\cdot T^{II}(t) \qquad or$$

$$\frac{c^2X^{II}(x)-k/X(x)}{X(x)}=\frac{T^{II}(t)}{T(t)} \qquad (4b)$$

The left part of this equality does not depend on t, the right - on x, therefore, their total value does not depend on either x or t, and therefore reduces to a constant, which we take in the form $-c^2\lambda^2$ (for $\lambda>0$). Then equation 4b decomposes into two ordinary differential equations:

$$T''(t)+c^2\lambda^2T(t)=0 \qquad (a)$$

$$c^2x''(x)-kX(x)+c^2\lambda^2X(x) \qquad (b)$$

Their solution (general integrals) have the form:
for a)

$$T(t)=Acos(c\lambda t)+Bsin(c\lambda t).$$

To solve equation b), the boundary conditions on the left and right ends should be taken into account:

$$\frac{\partial u}{\partial x}=0; \quad and \quad -k^I\frac{\partial u(x_2t)}{\partial x}=K_n^{II}(x_2,t)$$

$$c^2x''(x)+c^2\lambda^2X(x)=-K^Ix^I(x)=0$$

Solution of the equation:

$$c^2x''(x)+c^2\lambda^2X(x)=0$$

$$x''(x)+\lambda^2X(x)=0$$

Is:

$$X(x)=Ccosx+Dsin\lambda x$$

Then, for the condition on the free boundary $x=0$, we have:

$$X(x)=Ccos\lambda x+Dsin\lambda x$$

Or

$$Ccos\lambda x+Dsin\lambda x=0; \quad Ccos\lambda 0=0; \quad C=0$$

We consider the boundary condition for $x_2=1$, assuming that $C=0$

$$k_2sin\lambda x=-k_1cos\lambda x$$

or from where

$$k_2sin\lambda l=-k_1cos\lambda l;$$

$$tg\lambda l=-\frac{K_1}{K_2}\lambda=-\frac{K_1(\lambda l)}{K_2l} \qquad (5)$$

The graph shown in Fig. 1 provides information about the solutions of this equation. The positive roots $\lambda_1, \lambda_2 \ldots \lambda_n$ give us eigenvalues with corresponding eigenfunctions $sin\lambda x, sin\lambda_2 x, \ldots sin\lambda_n x$.

In other words, a number of values is obtained for λ

$$\lambda_n=-\frac{\xi}{l}$$

where
ξ (n are integers) are the positive roots of the transcendental equation:

$$tg\xi=-\frac{k1}{k2l}\xi$$

$$y=-\frac{k1}{k2}\lambda$$

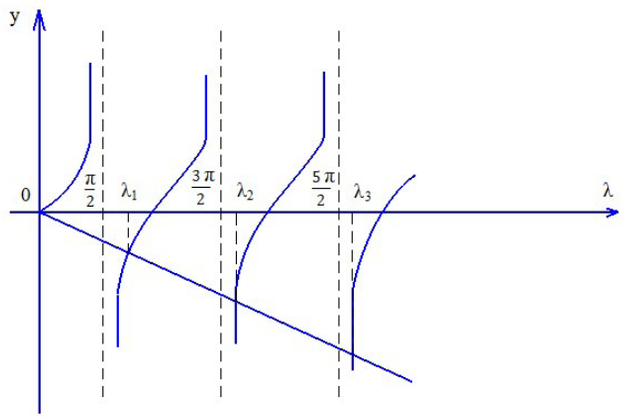

Fig. 1: Graphical definition of the positive roots of equation 5

For λ found in this way, in accordance with the superposition principle, we determine the general solution of the equation:

$$U(x,t)=\sum_{n=1}^{\infty}[Acos(c\lambda t)+Bsin(c\lambda t)]\,sin\lambda_n.$$

Made up of a countable number of solutions.

Assuming here that $c\lambda\omega$ $(\omega>0)$.

We will get

$$U(x,t)=\sum_{n=1}^{\infty}C_n\,sin(\omega_n t+\alpha)sin\lambda_n x \qquad (6)$$

We determine the coefficients of this series from the initial condition

$$U=f(x), \qquad (0\le x\le l)$$

Or

$$U=f(x)=\sum_{n=1}^{\infty}\overline{c_n}\,sin\lambda_n x=\sum_{n=1}^{\infty}\overline{c_n}\,sin\frac{\xi nX}{l}$$

4. CONCLUSION

The last equation can be considered as a generalized Fourier series in the interval (0, 1). Using the orthogonality of the found eigenfunctions

$$sin\frac{\xi nX}{l}$$

We determine the coefficients according to the known methods of normalization

$$C_n = \frac{\int_0^l f(x)\frac{\xi nX}{l}dx}{\int_0^l sin^2\frac{\xi nX}{l}dx}$$

For a known task ***f(x)***, these coefficients are uniquely found. The representation of the series in the form of 16 is an example of an anharmonic Fourier series (by the standing wave method). The components of this series do not have a common period, as in the harmonic Fourier series under normal conditions. The eigenvalues λn here have a bit more complex nature of their formation compared to those considerations when the boundary does not perform work, which is usually the case under other boundary conditions, starting from the initial moment of time ***(t=0)***.

Considering the expression 6, we see that the total oscillation of the rod ***u(x,t)*** is composed of a series of individual oscillations $u_n(x,t)$; where the points participating in such an elementary oscillation, determining the coordinates of the sections, oscillate with the same frequency. The amplitude of the oscillation of each point depends on its position. It is equal to:

$$\overline{C_n}\left|sin\frac{\xi nX}{l}\right|$$

The entire length of the rod is divided into not necessarily equal sections, and the point of the same section is always in the same phase, whereas the points of neighboring sections are in directly opposite phases. The points separating one section from another are at rest, these are the so-called 'nodes'. The midpoints of each of the sections ('antinodes') oscillate with the greatest amplitude, and, as can be seen from Fig. 1, the modes with a higher frequency of eigen-oscillations corresponding to the eigen-frequencies λ_i, 'fit" more closely to their corresponding vertical asymptotes.

Naturally, everything said about the rod I is exactly transferred to the rod II associated with it. When each of the rods vibrates at higher frequencies, there will be no multiplicity with respect to the main forms of vibrations, as it is easy to see in Fig. 1, for the boundary condition under consideration.

Since the steady-state energy mode of each of the rods is described by standing waves, as follows from the analysis of the solution, the average value of the energy flow for the period equals to zero, which allows us to conclude that there is no redistribution between neighboring antinodes in relation to kinetic and potential energy in a standing wave.

Taking further into account that at a steady energy state, a wave number is used as a characteristic of the description of the plane wave harmonicity, then with the known notation, on one side, we can write:

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{\mathrm{C}\cdot\mathrm{T}} = \frac{\omega}{C}$$

On the other side, based on the graphical construction for finding solutions, in accordance with Fig. 1, we establish that the roots of λ_n lie in the range:

$$\left(n-\frac{1}{2}\right)\pi < \lambda_n < \left(n+\frac{1}{2}\right)\pi$$
$$(2n-1)\frac{\pi}{2} < \lambda_n < (2n+1)\frac{\pi}{2}$$

Then for the eigen-frequencies we find the intervals of their changes taking into account

$$\lambda_n = \frac{\xi_n}{l} - \frac{\omega_n}{C}$$

We get the following:

$$\left(n-\frac{1}{2}\right)\pi < \frac{\omega_n l}{C} < \left(n+\frac{1}{2}\right)\pi$$
$$\frac{(2n-1)\frac{\pi}{2}C}{l} = \frac{(2n-1)\frac{\pi}{2}}{l}\left(\frac{E}{\rho}\right)^{\frac{1}{2}} < \omega_n < \frac{(2n+1)\frac{\pi}{2}}{l}\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$$

where
n is integers,
c is propagation velocity.

The results obtained in this paper can be extended to a rod system (which, as it was noted, serves as a research model), consisting of a finite number of rods, and interconnected by various coupling conditions.

In this case, the energy distribution law for such a system is represented in a similar form for the time $t \gg t^*_{max}$, where t^*_i is the time required to establish an equilibrium state for each of the system rods, i.e.

$$A = \sum_i^n E_i$$

Since during the wave process, the kinetic (***T***) and potential (***U***) energy in the considered volume of the medium reach their experimental values simultaneously, therefore, when averaging them over time for each of the rods, there is

$$\langle E_i \rangle = \langle T_i + U_i \rangle = U_i$$

assuming that in the volume of each of the rods under consideration, the average value of the total energy coincides with the maximum value of the potential energy.

On this basis, the last relation is represented as:

$$A = \sum_{i=1}^n U_i(x); \qquad (t > t^*_{maxi})$$

where
n is the number of elements included in the core system.

Studying the influence of internal losses on the energy distribution of sound vibration has two goals. Firstly, it is necessary to find out how losses affect the speed of establishing a stationary process in the system, and secondly, whether the system has a general loss coefficient in a stationary mode.

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