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THEORETICAL ANALYSIS OF SEMI-ACTIVE NOISE CONTROL

[a)]Thomas Kletschkowski

[a)]*University of Applied Science Hamburg, Germany, thomas.kletschkowski@haw-hamburg.de*

Abstract: *A significant reduction of disturbing noise can be achieved by passive, semi-active and fully active control approaches. Passive noise treatments such as dynamic vibration absorber are very robust and can be applied to obtain a broadband performance. Active noise control systems are designed to control harmonic or broadband noise. They are very effective, if the control volume is small as known from single-input/single-output systems used in active headphones. However, if distributed control is required, the control profit is not scalable, because the required multiple-input/multiple-output systems must be adjusted to specific acoustic modes as known from the active control of propeller-aircraft interior noise. Semi-active control that is based on the principle of dissipation allows to combine several single-input/single-output systems without coupling. Thus semi-active approaches are capable to solve the problem of scalability. The present paper reports on a specific approach that is based on a dynamic absorber attached to a vibrating structure and coupled with a dissipating electrical network. The electrical components of this network can be adjusted to the mechanical impedance to realize dissipation. To focus on the performance principle, the theoretical investigations are presented in a dimensionless analysis.*

Keywords: *Semi-active control, vibration absorber, dissipative network*

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1. INTRODUCTION

A significant reduction of airborne or structure-borne noise can be realized with passive, active or semi-active noise systems. A recapitulatory review on these approaches is given in [1].

Passive systems such as dynamic vibration absorber (DVA), compare [2], are very robust. They are usually designed to obtain a broadband performance, not only to neutralize the excitation force at a specific frequency. In order to realize this goal, it isessential to adjust the damping of the visco-elastic mount to an optimal value. DVA's are very robust. Because noise reduction is obtained by dissipation, several DVA's can be applied – without coupling – to increase the control profit. For this reason the performance is scalable and increased with the number of DVA's, the devices are not attached to nodal points.

In contrast to passive treatments active control systems, compare [3], are based on the principle of destructive interference. Thus, the excitation force is neutralized. This approach is very effective in the control volume is small as known from active headphones and single-input/single-output (SISO) control can be applied. If distributed control is required, multiple--input/multiple-output (MIMO) systems are required as known from the problem of active control of propeller-aircraft interior noise, compare [4]. As described in [3] the placement of sensors and actuators in such distributed control problems must be well chosen to ensure both observability and controllability. For this reason a MIMO active control systems is unique and not the result of a scaled approach based on multiple SISO systems. This is an up to now unsolved problem that prevents standardization and limits the application of distributed systems to a limited number of specific applications.

Semi-active noise treatments can be realized with variable stiffness and damping control, compare [5] and [6], or with stiffness-switch systems as reported in [7]. Furthermore concepts based on magnetorheological dampers, compare [8], and piezoelectric actuators, see [9], have been analyzed.

Another typical example is the electro-mechanical relaxation absorber (EMRA), as described in [10]. Such an EMRA can be understood as the combination of a DVA, attached to the surface a vibrating structure, and a dissipative network (DN) consisting of an electrical resistance in serial connection with an inductor. The DN is designed to dissipate the kinetic energy of the structure. The application of EMRA allows to solve the problem of scalability, because every single device contributes to the increase of damping. Furthermore it is possible to tune the connected DN to a specific application without changing the design of the underlying DVA. For these reasons the concept of noise control based on an EMRA is analyzed theoretically in the presented paper. To focus on the general performance principle, the investigations are presented in a dimensionless analysis. At first the equations of motion will be derived from a simplified model. In a second step, the noise control potential will be discussed. The paper ends with concluding remarks and a short outlook on future work.

2. A SIMPLIFIED MODELLING OF ELECTRO-MECHANICAL RELAXATION ABSORBER

2.1. Simplified model and equation of motion

As outlined in section 1 an EMRA consists of a DVA that is connected to an electric network. The DVA is attached to the vibrating structure via a visco-elastic mount. A simplified representation is shown in Fig. 1, where M_S represents the mass (SI-unit: ***kg***) of a vibrating surface, while M_S is the mass of the EMRA attached to reduce the vibration level, and thus the noise radiation. The stiffness of the structure is represented by K_S (SI-unit: ***N/m***). B_S represents structural damping (SI-unit: ***Ns/m***). The visco-elastic properties of the EMRA-mount are described by B_S and B_E. The force factor T (SI-unit ***N/A***) is used to model the electro-mechanical coupling. R is the electrical resistance (SI-unit: ***V/A***) and L is the inductance (SI-unit: ***Vs/A***) of the DN. The elements of the electro-mechanical network are assumed to have linear and time-invariant properties.

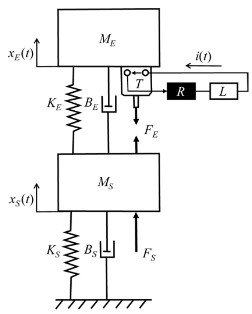

Fig. 1: Vibrating structure with semi-active vibration absorber

As also shown in Fig. 1., three coordinates have been introduced to describe the dynamics of the coupled system. These are: x_S (SI-unit: ***m***) – the displacement of the vibrating structure, x_E – the displacement of the DVA x_S, and i (SI-unit: ***A***) – the electric current in the DN that is later on replaced by the electric charge q (SI-unit: ***As***) considering $i = dq/dt$. All coordinates are time-dependent variables depending on the physical time t (SI-unit: ***s***). The excitation force F_s (SI-unit: ***N***) is introduced to model the noise source. Static deflection has not been taken into account.

Applying the laws of NEWTON and KIRCHHOFF it is possible to derive the set of equations of motion represented by Eqn. (1) – (3). These are ordinary differential equations of second order depending on time. All equations are linear. Because all elements are time-invariant, all nine coefficients are constant in time. The right-hand-side is determined by the excitation force.

$$M_S\frac{d^2x_S}{dt^2}+(B_S+B_E)\frac{dx_S}{dt}-B_E\frac{dx_E}{dt}+T\frac{dq}{dt}+(K_S+K_E)x_S-K_Ex_E=F_S \tag{1}$$

$$M_S\frac{d^2x_E}{dt^2}-B_E\frac{dx_S}{dt}+B_E\frac{dx_E}{dt}-T\frac{dq}{dt}-K_Ex_S+K_Ex_E=0 \tag{2}$$

$$L\frac{d^2q}{dt^2}-T\frac{dx_S}{dt}+T\frac{dx_E}{dt}+R\frac{dq}{dt}=0 \tag{3}$$

2.2. Dimensionless formulation of system equation

To reduce the complexity and to focus on the general performance principle, the equations of motions are transformed to a non-dimensional representation of the problem. For this reason, the abbreviations listed in Eqn. (4) have been introduced.

$$w_0^2:=\frac{K_S}{M_S}\qquad 2Dw_0:=\frac{B_S}{M_S}\qquad 2Ew:=\frac{R}{Lw_0} \tag{4}$$

w_0 is an angular frequency defined by the ratio of stiffness and mass of the vibrating structure.
D is the dimensionless structural damping ratio and
E is the dimensionless damping ratio of the DN. The latter is introduced as the ratio between electrical resistance and inductance normalized to w_0.

Furthermore, relative values for the remaining elements have been introduced, compare Eqn. (5).

$$b:=\frac{B_E}{B_S}\qquad g:=\frac{T^2}{RB_S}\qquad m:=\frac{M_E}{M_S}\qquad n:=\frac{K_E}{K_S} \tag{5}$$

b is a relative viscosity obtained by normalizing the viscosity of the DVA-mount to the viscosity of the vibrating structure. g can be interpreted as a dimensionless force factor. However it represents a mechanical viscosity, given by T^2/R, that is normalized by using the viscosity of the vibrating structure.
m is the relative mass and
n the stiffness ratio.
To proceed it is also necessary to introduce the generalized displacement
x_Q as defined its time derivative, compare Eqn. (6)

$$\frac{dx_Q}{dt}:=(T/B_S)\frac{dq}{dt} \tag{6}$$

Based on w_0 the dimensionless time r can be introduced as well as the corresponding derivatives. These definitions are given in Eqn. (7)

$$r:=w_0t\qquad \frac{d(.)}{dt}=\frac{d(.)}{dr}\qquad \frac{dr}{dt}:=\frac{w_0d(.)}{dr}\qquad \frac{d^2(.)}{dt^2}=\frac{d^2(.)}{dr^2}\qquad \frac{d^2r}{dt^2}:=\frac{{w_0}^2d^2(.)}{dr^2} \tag{7}$$

Finally it is necessary to introduce dimensionless coordinates representing the degrees of freedom for structure (y_S), DVA (y_E), and DN (y_Q) as well the dimensionless excitation force f_s, compare Eqn. (8). x_0 is an arbitrarily chosen factor and used to normalize the physical coordinates.

$$y_S:=\frac{x_S}{x_0}\qquad y_E:=\frac{x_E}{x_0}\qquad y_Q:=\frac{x_Q}{x_0}\qquad f_S:=\frac{F_S}{Lw_0^2x_0} \tag{8}$$

The resulting set of dimensionless equations of motion is summarized in Eqn. (9) – (11). It is worth mentioning that the number of constants is reduced from nine, compare Eqn. (1) – (3), down to six, if the problem is analysed in a dimensionless formulation. This is important, if optimization is used in the future to find an optimal set of parameter.

$$\frac{d^2y_S}{dr^2}+2D(1+b)\frac{dy_S}{dr}-2Db\frac{dy_E}{dr}+2D\frac{dy_Q}{dr}+(1+n)y_S-ny_E=f_S \tag{9}$$

$$m\frac{d^2y_E}{dr^2}-2Db\frac{dy_S}{dr}+2Db\frac{dy_E}{dr}-2D\frac{dy_Q}{dr}-ny_S+ny_E=0 \tag{10}$$

$$\frac{d^2y_Q}{dr^2}-2Ewg\frac{dy_S}{dr}+2Ewg\frac{dy_E}{dr}-2Ew\frac{dy_Q}{dr}=0 \tag{11}$$

Assuming time-harmonic fluctuations of all quantities according to the normalized frequency f given by the ratio of angular excitation frequency W and w_0, compare Eqn. (12)

$$f := \frac{W}{w_0} \tag{12}$$

the dimensionless complex compliance is given by Eqn. (13)

$$\{C\} = (-f^2\{M\} + jf\{B\} + \{K\})^{-1} \tag{13}$$

where *{M}* is the dimensionless mass matrix, *{B}* is the dimensionless damping matrix, and *{K}* is the dimensionless stiffness matrix as defined in Eqn. (14) – (16).

$$\{M\} = [[1,0,0];[0,m,0];[0,0,1]] \tag{14}$$

$$\{B\} = [[2D(1+b),-2Db,2D];[-2Db,2Db,-2D];[-2Ewg,2Ewg,2Ew]] \tag{15}$$

$$\{K\}=[[1+n,-n,0];[-n,n,0];[0,0,0]] \tag{16}$$

The dynamic behaviour of the system is described by nine complex and dimensionless frequency response functions C_{ij}. The index i represents the response of the ***i***-th coordinate due to excitation at the ***j***-th position. In order to study the system response it is possible to introduce a logarithmic measure as defined in Eqn. (17).

$$H_{ij} := 20\log_{10}(|C_{ij}|) \tag{17}$$

3. NUMERICAL INVESTIGATION OF NOISE CONTROL POTENTIAL

In order to analyze the noise control potential of an EMRA a specific set of parameter listed in Tab.1 have been defined. It represents a structure with low structural damping and a DVA with a nearly elastic mount. Thus, the relevant dissipation is associated with the DN. The value of ***Ew*** that can be interpreted as the normalized decay constant of electrical circuit as well as the values for ***g*** – the normalized force factor – have been defined according to small common-of-the-shelf vibration exciter known from flat panel speaker systems. For this reason the numerical example represents a system similar to a composite lightweight structure, as used for aircraft interior design, with point force excitation in combination with a semi-active noise reduction treatment based on an EMRA. The simulation results are shown in Fig. 2, where H_{11} represents the magnitude response of the structure due to an excitation acting on the structure, H_{12} represents the magnitude response of the DVA due to an excitation acting on the structure, and H_{13} represents the magnitude response of the DN due to an excitation acting on the structure.

Parameter	Symbol	Value
Damping ratio of structure	*D*	0,005
Relative mass	*m*	0,100
Relative damping	*b*	0,100
Relative stiffness	*n*	0,100
Normalized decay constant of electrical circuit	*Ew*	456,9
Normalized force factor	*g*	[0; 0,5; 1.5; 2.5; 3.5; 4.5]

Tab. 1: Parameter used to describe the system dynamics

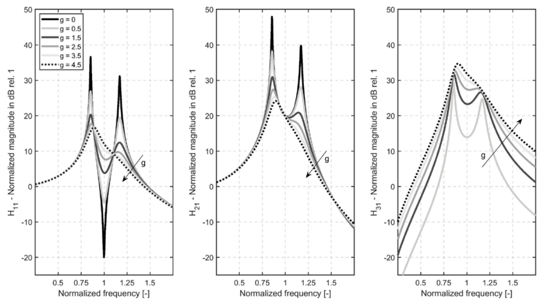

Fig. 2: Normalized magnitudes of structure (left), absorber (middle), and electric circuit (right)

As indicated by the last row in Tab. 1 and shown in Fig. 2 different vales of the normalized force factor ***g*** have been used to simulate the frequency response of the electro-mechanical system shown in Fig. 1. According to the definition of ***g***, compare Eqn. (5), two interpretations are possible for this procedure

- **Interpretation 1:** A variation of ***g*** corresponds to a proportional variation of ***T*** and can be seen as an adaption of the DVA. Compared to active control systems this could be understood as an adaption of the actuator system in order to generate an effective actuation force.
- **Interpretation 2:** A variation of ***g*** corresponds to an indirect proportional variation of ***R*** and can be seen as an adaption of the DN. Compared to active control systems this could be understood as an adaption of the control schema without changing the actuation system. This refers to the scaling problem outlined in section 1, because the same DVA is used, but the performance of the DN is scaled (better adapted) to obtain an effective dissipation.

Naturally it is also possible to analyze a variation of the normalized decay constant ***Ew*** that represents the ratio of ***R*** and ***L***. However, interpretation 1 would have been impossible following this approach.

The situation without DN corresponds to the value $g = 0$. The resonance frequencies are to be found at $f_{R1} = 0{,}86$ and $f_{R2} = 1{,}17$. The anti-resonance is (in agreement with the theory of dynamic absorbers) to be found at $f_{AR} = 1{,}00$. The control profit at the resonance frequencies increased with an increase of ***g***, compare Fig. 2 (left) and Fig. 2 (middle). At the same time the normalized electric charge in the DN increases too as shown in Fig. 2 (right). Furthermore the effect of neutralizing the excitation decreases, if ***g*** increases. The results proof that an optimal value for ***g*** could be found between $g = 3{,}5$ and $g = 4{,}5$. Further increase of the normalized force factor would result in a strongly coupled system and an increase of the system response at f_{AR}.

Frequency	g = 0,5	g = 1,5	g = 2,5	g = 3,5	g = 4,5
1st resonance in H_{11}	9,40	16,4	18,9	19,5	20.2
1st resonance in H_{21}	9,90	17,9	20,7	22.8	23.9
1st resonance in H_{31}	-29,3	-31,6	-32,8	-33,5	-34,6
Anti-resonance in H_{11}	-15,9	-23,8	-27,7	-30,0	-31,5
Anti-resonance in H_{21}	0,00	0,00	0,00	0,00	0,00
Anti-resonance in H_{31}	-13,9	-23,2	-27,4	-30,0	-32,2
2nd resonance in H_{11}	11,6	18,7	21,4	22,7	23,4
2nd resonance in H_{21}	11,4	19.4	21,8	23,8	25,1
2nd resonance in H_{31}	-25,0	-26,6	-27,4	-28,8	-28,9

Tab. 2: Development of the control profit in dB and induced charge in dB compared to ***g = 0***.

The evolution of control profit at the resonance frequencies f_{R1} and f_{R2}, its decrease at the anti-resonance f_{AR} as well as the increase of the electric charge in the DN are shown in Tab. 2. The control profit is the difference between the magnitude at the analyzed frequency lines for ***g = 0,0*** and whose values obtained for the same frequency lines and positive values of ***g***. Thus, a positive control profit represents a noise reduction, while negative values are associated with an increase of the vibration level. The same holds for the electric charge.

The data shown in Tab. 2 proof that a significant amount of noise reduction is possible also for the smallest value of the normalized force factor, because a vibration control profit of nearly ***10 dB*** is realized for ***g = 0,5*** at f_{R1}. At f_{R2} the vibration control profit is approximately ***11,5 dB***. At the same time a significant attenuation remains for H_{11} at f_{AR}. The vibration control profit increases significantly, if the normalized force factor increases. It is worth notifying that the increase of the electric charge is lower comparted to the increase of the vibration control profit. The data also proof that a saturation is reached for higher values of ***g***. The highest noise reduction for the response of the structure associated with a control profit of ***23.9*** is obtained for H_{21} at ***g = 0,5***.

4. CONCLUSION

Theoretical investigations on semi-active noise control based on EMRA have been presented. A simplified lumped model have been proposed to analyze the concept.

It has been shown that a normalized and dimensionless formulation is advantageous because the number of parameter that have to be analyzed can be reduced. Furthermore it has been shown that the concept of semi-active control based on EMRA can lead to different engineering problems. If the vibrating structure is given and the DN is already designed, it is necessary to adjust the DVA in order to realize a significant control profit. In this case especially a well-chosen force factor has to enable an appropriate coupling behavior between DVA and DN. If the vibration structure is given and the design of the DVA is completed, the engineering problem is given by the design of the DN.

The second problem also explains the scalability of the approach, because the same DVA can be applied to different structures and only the DN has to be re-designed. This could be a great benefit for product development. However, semi-active control based on EMRA also allows a scalability because several single EMRA can be applied – without cross coupling – to increase the control profit. This benefit results from the fact that the approach is based on energy dissipation and not on destructive interference as known from active noise control approaches.

Future work will be focused on numerical as well as on theoretical studies in order to prepare practical applications. Especially new engine concepts in airplane engineering could lead to applications in which interior noise must be controlled without additional power consumption. For this reason future investigations will be carried out in order to analyze the noise control potential of an EMRA-approach applied to lightweight composite structure used to cabin interior wall.

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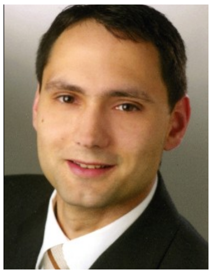

Thomas Kletschkowski *is Doctor of Engineering Science with Habilitation in the field of Mechatronics, Professor, Head of the Department for Automotive and Aeronautical Engineering of the HAW Hamburg University of Applied Sciences. Thomas Kletschkowski is a specialist in non-linear continuum mechanics and Mechatronics with specialization in active control of sound and vibration. He is a member of the editorial board of the Akustik Journal that is published by the German Society for Acoustics (DEGA). Thomas Kletschkowski is the author of over 100 scientific publications. He presented the main results of scientific research at international conferences in Austria, Italy, the Netherlands, Sweden, Poland, UK, USA, and Germany.*