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LOW-FREQUENCY IMPEDANCE OF ANISOTROPIC ENGINEERING DESIGN

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***Abstract**: The results of theoretical and experimental research of the impedance of an anisotropic structure in the low frequency range are presented. It is shown that taking into account the inhomogeneous distribution of mass along the length of the structure has a great influence on the value of the impedance of an anisotropic structure. Based on the results obtained in the mathematical model, expressions are proposed for calculating the impedances of anisotropic structures in the low frequency range.*

***Keywords:** Mathematical model, experiment, low-frequency oscillations, input impedance, transition impedance, mass impedance, anisotropic structure, homogeneous structure*

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1. INTRODUCTION

Impedance characteristics of engineering structures are widely used in solving a wide range of vibroacoustic problems [1–18], e.g., such as:

- radiation of engineering structures to the environment [8, 10, 12, 16];
- vibration damping in engineering structures [1–4, 8, 10, 11, 13, 15–18];
- isolation of low-frequency vibrations of mechanisms [5, 8, 9, 14, 16, 18];
- the functioning diagnostics of engineering structures [6, 7, 15].

Depending on the excitation frequency, an engineering structure freely suspended oscillates in the low-frequency range as a solid whole and as the frequency increases, resonant vibration arises in it.

At present, when considering low-frequency oscillations of engineering structures as a solid whole, the latter is represented as a homogeneous beam with a mass uniformly distributed along its length. In [8], expression (1) is given. This expression allows calculating the impedance value Z_{hw} of a freely suspended structure in relation to the driving force applied at any point along the length of a homogeneous element.

$$Z_{hw} = \frac{j\omega M}{1-3\left(\frac{2x_a}{l}-1\right)\left(1-\frac{2x_F}{l}\right)}. \tag{1}$$

where
M is the mass of a homogeneous element,
$\omega=2\pi f$ is the circular frequency,
x_a is the distance from the observation point to the end of the element,
x_F is the distance from the point of the force application to the end of the element,
l is the length of the element.

A comparative analysis of the experimental data obtained for heterogeneous structures (casings with a set of stiffeners, casings with equipment installed in them, etc.) and the calculated estimates obtained by expression (1) has shown that the calculated estimates differ significantly from the experiment.

Therefore, expression (1) cannot be used to calculate the impedance magnitude of anisotropic structures in the low frequency range.

The purpose of this article is to obtain an analytical dependence for calculating the impedance characteristic of an anisotropic engineering structure in the low-frequency range.

2. MATHEMATICAL MODEL

The necessary expression for calculating the impedance magnitude of anisotropic structures in the low-frequency range can be obtained as follows. Constructively, an anisotropic structure in the pre-resonant frequency range $0 < f < f1$ ($f1$, Hz – frequency starting from which the oscillating system cannot be considered as a solid whole) can be represented as a beam with a center of mass arbitrarily located relative to the axis of symmetry. Consider a mathematical model describing a beam with an arbitrarily located center of mass. The beam is installed on two vibration isolators with the same stiffness and at a point with a coordinate x_F, harmonic driving force $F\cos(\omega \cdot t)$ ($\omega=2\pi f$ – circular frequency) affects the beam (Fig. 1). Oscillations of such a system were researched in a number of papers, e.g., in [5, 9] and can be described by a system of differential equations

$$\left.\begin{aligned} M\frac{dv_M}{dt} &= R_B + R_A + F\cos(\omega t) \\ J\frac{d\varphi}{dt} &= bR_B + aR_A + x_F F\cos(\omega t) \\ R_A = -c\frac{d\Delta y_A}{dt};\ & R_B = -c\frac{d\Delta y_B}{dt} \\ v_A = \frac{d\Delta y_A}{dt};\ v_B = \frac{d\Delta y_B}{dt};\ & v_A = v_M - \varphi a \\ v_B = v_M + \varphi b;\ & v_X = v_M + \varphi x \end{aligned}\right\} \qquad (2)$$

where:

M, J – mass and moment of inertia of the beam mass relative to the center of mass;,

R_A, R_E – the reaction of vibration isolators at points A and B; F,

ω – the amplitude and circular frequency of the driving force;

c – the stiffness coefficient of each of the vibration isolators;

V_A, V_B, V_M, V_X– the speed of the left and right vibration isolators, the center of mass and the beam point with the coordinate ***x***, respectively;

φ – the angular velocity of torsional vibrations of the structure; ***a, b*** – the distance from the left support A to the center of mass of the structure and from the center of mass of the structure to the right support B (Fig. 1);

X_F, X – the coordinates of the points where the oscillations are excited by the driving force ***Fcos(ω·t)*** and the oscillation rate is measured, respectively;

Δy_A, Δx_B – dynamic movements of the right and left vibration isolators, respectively.

After transformations, the system of differential equations (2) will be written in the form

$$\left.\begin{aligned} M\frac{dv_X}{dt} - Mx\frac{d\varphi}{dt} - (R_B + R_A) &= F\cos(\omega t) \\ J\frac{d\varphi}{dt} - (bR_B - aR_A) &= x_F F\cos(\omega t) \\ -\left(\frac{dR_B}{dt} + \frac{dR_A}{dt}\right) &= c[2v_X + (b-a-2x)\varphi] \\ -\left(b\frac{dR_B}{dt} - a\frac{dR_A}{dt}\right) &= c\{(b-a)v_X + [a^2+b^2-(b-a)x]\} \end{aligned}\right\} \qquad (3)$$

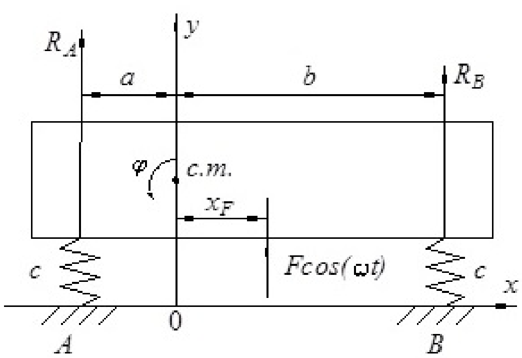


Fig. 1: Mathematical model of a vibroinsulated structure with a displaced center of mass (c.m.

After the transition to a complex form of recording, one will seek a particular solution of this system in the form of steady-state oscillation: $v_x = \bar{v}_x e^{j\omega t}$, $\varphi = \bar{\varphi}e^{j\omega t}$.

Then from system (3) we obtain the following algebraic equations with unknowns $\bar{v}_x$ and $\bar{\varphi}$

$$\left.\begin{aligned} \left(j\omega M + \frac{2c}{j\omega}\right)\overline{v_X} + \left[\frac{c}{j\omega}(b-a-2x) - j\omega Mx\right]\bar{\varphi} &= F \\ \frac{c}{j\omega}(b-a)\overline{v_X} + \left\{\frac{c}{j\omega}\left[a^2+b^2-(b-a)x\right] + j\omega J\right\}\bar{\varphi} &= x_F F \end{aligned}\right\} \qquad (4)$$

From equations (4), the complex velocity $\bar{v}_x = \bar{v}(x)$is expressed as

$$\overline{v_X} = \frac{\begin{vmatrix} D_1 & D_2 \\ D_3 & D_4 \end{vmatrix}}{\begin{vmatrix} D_5 & D_6 \\ D_7 & D_8 \end{vmatrix}} F \qquad (5)$$

where

D_1=1;

D_2=c/(jω) (b-a-2x)-jwMx;

D_3=xF;

D_4= c/(jω) [a^2+b^2-(b-a)x]+ jJ;

D_5= ωM+2 c/(j ω); D_6=D_2;

D_7=c/(jω)(b-a);

D_8=D_4.

Expression (5) determines the transition impedance $\mathbf{Z}_{per}$ of the system at a point with a coordinate x in relation to the force ***Fe^{jwt}*** applied at the point ***x_F*** as $\mathbf{Z}_{per}$=**F/$\overline{v_X}$**, or after carrying out algebraic transformations of determinant (5):

$$Z_{per} = \frac{j\omega M}{1+MX^{x_F}/J}K(\omega;x) \qquad (6)$$

Where

$$K(\omega;x) = \frac{1 - \dfrac{c}{c^2 M}\left[2 + M\dfrac{b^2+a^2}{J} + c(a+b)\omega^2 J\right]}{1 - \dfrac{c(b^2+a^2)}{\omega^2(J+Mxx_F)}\left[1 - \dfrac{(b-a)(x+x_F)}{b^2+a^2} + \dfrac{2xx_F}{b^2+a^2}\right]}$$

When

x=x_F , expression (6) determines the input impedance of the structure.

Analysis of expressions (6) shows that the frequency response of a mechanical impedance has two resonances and one antiresonance. The proper frequencies ***$\omega^{(r)}_{1,2}$*** of this system are defined as

$$\omega^{(r)}_{1,2} = \left[1 \pm \left(1 - \frac{Q}{P^2}\right)^{0.5}\right]^{0.5} P^{0.5} \qquad (7)$$

where

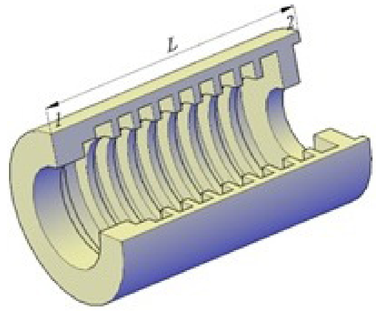
$P = c/M\left[1 + M\left(a^2+b^2\right)/(2J)\right]$

$Q = c^2(a+b)^2/(JM)$, and the antiresonance frequency ***$\omega^{(ar)}$*** as

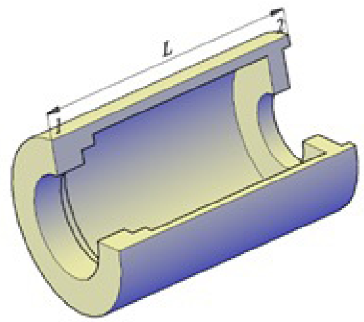
$$\omega^{(ar)} = \left\{\left[\frac{c(a^2+b^2)}{(J+Mxx_F)}\right]\left[1 - \frac{(b-a)(x+x_F)}{(a^2+b^2)} - \frac{2xx_F}{a^2+b^2}\right]\right\}^{0.5} \qquad (8)$$

According to expression (6), a calculation for a mathematical model that is a structure in the form of model 1 shown in Fig. 2 a) and installed on two identical vibration isolators was made.

The ratio of stiffness of shock absorbers ***c*** to the casing mass ***M*** is ***c/M=150s²***. The structure was excited in the radial direction by a harmonic force at point 2 at a distance x_F from the center of mass of the object (Fig. 2 a).



(a)



(b)

Fig. 2: Design of models experimentally studied

The results of calculations in the form of a dependence ***|Z|/(ωM)*** on the excitation frequency f are presented in Fig. 3. Curve 1 shows option 1 of installing the casing on the vibration isolators when they were placed along the edges of the casing, and curve 2 – option 2, in which the left vibration isolator was moved to a point located at a distance of 0.01 meters to the left of the structure center of mass. Curve 3 obtained experimentally with free suspension of model1 on strings is also plotted here.

Analysis of the measurement results and theoretical estimates for this and other cases shows a good agreement between the results in the low-frequency region investigated. Moreover, it was found that the influence of the stiffness and placement of vibration isolators on the measured impedance value is noticeable up to the boundary frequency $f_{gr} \leq 2/\pi(2c/M)^{-0,5}$. At higher frequencies, up to the frequency f_1 , which restricts the possibility of representing the structure as a model of a single solid body one can use the model of a freely suspended body.

Thus, in the frequency range $ff_{gr} \leq \leq f_1$, expression (6) is simplified to the form

$$Z_{per} = \frac{j\omega M}{1 + Mx^{x_F}/J} \qquad (9)$$

describing the impedance of inhomogeneous freely suspended structures, in particular, axisymmetric casings with an uneven mass distribution along the length of the casing (ribs, connecting rings, fixed devices, and so on).

3. EXPERIMENTAL DATA

Experimental verification of the results obtained above was carried out on two kinds of models shown in Fig. 2. Model 1 is a cylindrical casing supported by stiffeners and massive rings located at the edges, model 2 is a cylindrical shell with massive rings located at the edges.

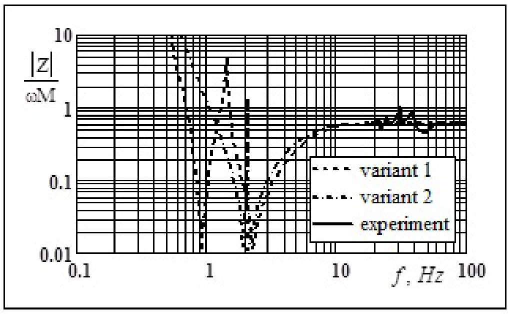
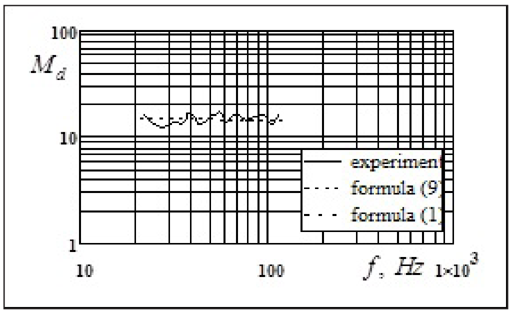


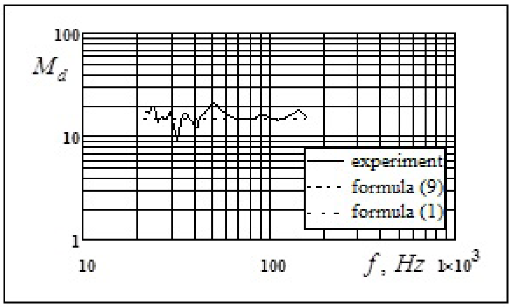
Fig. 3: Input dynamic mass module of model 1 at point 2

Fig. 4 shows the results of experimental determination and theoretical calculation of the input dynamic mass module M_d= ***|Z/ω|*** of model 1 (Fig. 3 a) when excited by a harmonic force at points 1 (Fig. 4 a) and 2 (Fig. 4 b), respectively. Formulas (1) and (9) have been used.

Fig. 5 shows the results of experimental determination and theoretical calculation of the input dynamic mass module M_d= ***|Z/ω|*** of model 2 (Fig. 3 b) when excited by a harmonic force at points 1 (Fig. 5 a) and 2 (Fig. 5 b), respectively. Formulas (1) and (9) have been used.



(a)



(b)

Fig. 4: Input dynamic mass module of model 1 at frequencies ***ff≤***$_1$

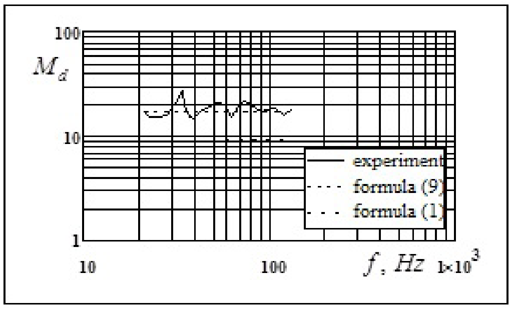
In order to determine the effect of inhomogeneity in mass distribution, the same figures show curves 3 corresponding to the calculation of the low-frequency impedance according to expression (1).

If a structure with a uniform arrangement of mass along the length occurs, then expression (9) will take the form

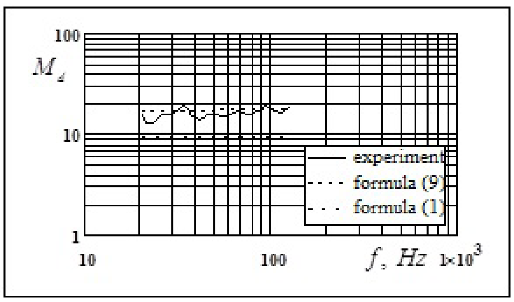
$$Z_{per}^{(odn)} = \frac{j\omega M}{1+12x^{x_F}/_{l^2}} \quad (10)$$

Note that accurate to specifying the origin of coordinates ***x*** and ***x***

Note that accurate to specifying the origin of coordinates ***x*** and $\boldsymbol{x_F}$, expression (10) coincides with expression (1) given in [8] for a freely suspended homogeneous beam (where instead of ***x*** and $\boldsymbol{x_F}$, coordinates ***(x-1/2)*** and $\boldsymbol{(x_F\text{-}1/2)}$ corresponding to the calculation of coordinates from the left edge of the beam are used).



(a)



(b)

Fig. 5: Input dynamic mass module of model 2 at frequencies ***ff≤***$_1$

Comparison of the results of calculations by expressions (9) and (1) with the data of the experimental determination of impedance characteristics shows that for the investigated casing structures, the calculation by formula (9) is in good agreement with experiment while the results of calculations by formula (1) or (10) not taking into account the mass distribution heterogeneity differ from the experimental values by about 5–6 dB.

This is due to the difference in the intensity of the inertial resistance of a homogeneous and inhomogeneous structure to torsional vibrations, which are excited along with reciprocating vibrations when excited by a force at a point that does not coincide with the center of mass of the structure.

In the investigated casing structures, the mass distribution was such that the moment of inertia of each of them was greater than that of a homogeneous structure equivalent in mass and dimensions. Therefore, their impedance being measured exceeded that for equivalent homogeneous bodies.

This is additionally illustrated in Fig. 6, where the curves of the calculation values of the input impedance module normalized to the value ***ωM*** in the frequency range $\boldsymbol{ff_{gr}} \leq \leq \boldsymbol{f_1}$ are plotted when the point of application of the driving force along the generatrix of model 2 from its left edge to the right is displaced. The only common point between these two curves is the point corresponding to excitation to the center of mass. At all other points, the calculation estimates diverge the more significantly, the further from the center of mass the excitation point is located.

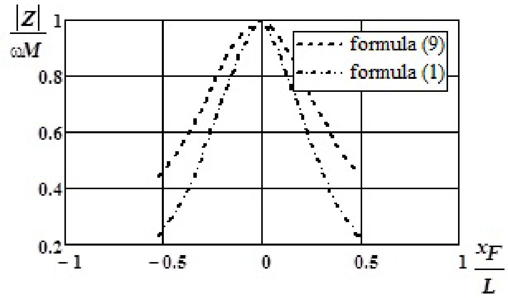


Fig. 6: Model of the input dynamic mass depending on the point of force application along the length of model 2

4. CONCLUSIONS

Considering the above, the following conclusions can be drawn:

1. 1. The developed computational model (9) for determining the low-frequency transition and input impedances in a structure with an inhomogeneous mass distribution has a sufficient degree of adequacy in the specified frequency range, which is confirmed by the agreement of the calculated and experimental characteristics of a number of casing structures.
2. The proposed model:
   - allows determining the lower limit of the frequency range where you can experimentally determine the impedance of the structure installed on vibration isolators without fear of distortion caused by the influence of the latter;
   - can be useful both for evaluating the reliability of experimentally obtained impedance characteristics of anisotropic structures, and for making theoretical estimates of the mechanical impedances of such structures in the frequency range .
3. The research carried out using the constructed computational model have shown that in the indicated low-frequency region for casing structures, the inhomogeneity of the mass distribution significantly affects the magnitude of both the input, and transient mechanical impedance.

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