OPTIMUM SOUND PROPAGATION IN OCEAN: THE COMPARISON BETWEEN NORMAL MODE AND EMPIRICAL FRANCOIS-GARRISON FORMULA

Tran Cao Quyen
Faculty of Electronics and Telecommunications, University of Engineering and Technology, Hanoi Capital, Vietnam. Email: quyentc@vnu.edu.vn

Abstract: There are not only theory but also the empirical formula to deal with the problem of sound propagation in ocean. As such the Normal mode and Francois-Garrison are typical methods. Optimum sound propagation in ocean is the way we unify these two methods. In this paper, we investigate the transmission losses of both Normal mode and Francois-Garrison at some frequencies (170Hz, 800Hz, 100 KHz) at shallow water conditions (Tonkin gulf). The calculated and simulation results show that the optimum is achieved when the Normal mode is used at low frequencies whereas the Francois-Garrison is used at high frequencies.

1. Introduction

The problem of underwater sound propagation take an important role to SONAR technique. The applications of SONAR are but not limited to submarine, floating boat detection and localization in military section or fish finding, sea bed supervision, drill surveillance in civil industry.

In term of the theory of underwater sound propagation, the Normal mode is considered as a worldwide acoustic community [1-5]. In this theory, the ocean environment likes an oceanic wave guide which is limited by sea surface and sea bed. The received acoustic pressure is sum of propagation modes in the oceanic wave guide with horizontal propagation constant Krm or is called the eigenvalue. The transmission loss is due to evanescent modes (leakage) in the sea bed layers and geometric spreading loss (cylindrical or spherical spreading).

In term of empirical ocean measurements, regarding geometric spreading loss, the underwater sound absorption caused by the three main factors, i.e., Boric acid, Magnesium sulfate and pure water. It is the Francois-Garrison empirical formula [6-10]. This result is come from many measurements from different atlantics, temperature, depth and pH.

This paper is a comparison between the transmission loss of Normal mode theory and that of the Francois-Garrison empirical formula for low frequencies less than 1000 Hz (170 Hz, 800 Hz) and 100 KHz for high frequency.

The comparison shows that there is a consistent between transmission losses of Francois-Garrison and the Normal mode at low frequencies (170 Hz, 800 Hz<1000 Hz). However, at high frequency the transmission loss of Francois-Garrison is much higher than that of Normal mode. The reason for this non-consistent may be in Normal mode theory one considered that the water column is homogeneous for all acoustic vibrations, it is not true in reality. This finding is very important for the precisely calculation of underwater sound propagation. We may conclude that for optimum underwater sound propagation at low frequencies the Normal mode is preferred and at high frequencies the empirical formula of Francois-Garrison is used.

The rest of the paper is organized as follows. In Part 2 the Normal mode theory is presented. Part 3 describes the empirical sound absorption of Francois-Garrison. The comparisons of the transmission losses between Normal mode and the empirical approach are given in Part 4. Finally, we concludes the paper in Part 5.

2. Transmission loss of Normal mode

Staring from Helmholtz equation in two dimensions with sound speed c and density ρ depending only on depth z [1-5]

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})+\rho(z)\frac{\partial}{\partial z}(\frac{1}{\rho(z)}\frac{\partial\psi}{\partial z})+\frac{\omega^2}{c(z)^2}\psi=-\frac{\delta(r)\delta(z-z_s)}{2\pi r} \quad (1)$$

where z_s is source depth, z is depth and r is distance.

Using separation of variables , we obtain the $\psi(\mathrm{r,z})=\Phi(\mathrm{r}).\mathrm{V(z)}$ modal equation

$$\rho(z)\frac{d}{dz}[\frac{1}{\rho(z)}\frac{dV_m(z)}{dz}]+[\frac{\omega^2}{c(z)^2}-k_{rm}{}^2]V_m(z)=0 \quad (2)$$

with the boundary conditions such as

$$V(0)=0,\frac{dV}{dz}\Big|_{z=D}=0 \quad (3).$$

The former condition implies a pressure release surface and the latter condition is from a perfect rigid bottom. The modal equation that is the center of the NM, has an infinite number of modes. Each mode represents by a mode amplitude $V_m(z)$ and a horizontal propagation constant k_{rm}. $V_m(z)$ and k_{rm} are also called *eigenfunction* and *eigenvalue* respectively

Noting that the modes are orthonormal, i.e.,

$$\int_0^D \frac{V_m(z)V_n(z)}{\rho(z)}dz=0, \quad \mathrm{m}\neq\mathrm{n}$$
$$\int_0^D \frac{V_m(z)^2}{\rho(z)}dz=1 \quad (4)$$

Since the modes forms a complete set, the pressure can represents as a sum of the normal modes

$$\psi(r,z)=\sum_{m=1}^{\infty}\Phi_m(r)V_m(z) \quad (5)$$

After some manipulations, we obtain

$$\psi(r,z)=\frac{i}{4\rho(z_s)}V_m(z_s)H_0^1(k_{rm}r) \quad (6)$$

where H_0^1 is the Hankel function of the first kind.

Substitute (6) back to (5) we have

$$\psi(r,z)=\frac{i}{4\rho(z_s)}\sum_{m=1}^{\infty}V_m(z_s)V_m(z)H_0^1(k_{rm}r) \quad (7)$$

Finally, using the asymptotic approximation of the Hankel function, the pressure can be written as

$$\psi(r,z)\approx\frac{i}{\rho(z_s)\sqrt{8\pi r}}e^{-i\pi/4}\sum_{m=1}^{\infty}V_m(z_s)V_m(z)\frac{e^{ik_{rm}r}}{\sqrt{k_{rm}}} \quad (8)$$

3. Transmission loss of Francois-Garrison

There are some empirical methods to deal with the problem of sound propagation in ocean such as [6-10]. However, we interested in Francois-Garrison since they used many measurements from different atlantics, temperature, depth and pH.

According to [6-7] the total sound absorption in ocean consists of three main components. i.e., Boric acid, magnesium sulfate and pure water.

$$\textit{Total absorption} = \textit{Boric Acid} + MgSO_4 + \textit{Pure Water}$$

$$\alpha=\frac{A_1P_1f_1f^2}{f_1^2+f^2}+\frac{A_2P_2f_2f^2}{f_2^2+f^2}+A_3P_3f^2 \quad (9)$$

Where f is acoustic frequency, f_1, f_2 are relaxation frequencies of boric acid and magnesium sulfate, P_1, P_2, P_3 are nondimensional pressure correction factors.
Boric Acid contribution

$$A_1=\frac{8.86}{c}x10^{(0.78\mathrm{pH}-5)} \quad dBkm^{-1}kHz^{-1}$$
$$P_1=1$$
$$f_1=2.8(s/35)^{0.5}10^{(4-1245/\theta)}$$
$$c=1412+3.21T+1.19S+0.0167D \quad (10)$$

C is sound speed, T is temperature in degree, S is salinity ($‰$), D is the depth (m), $\theta=273+T$
$MgSO_4$ contribution

$$A_2=21.44\frac{s}{c}(1+0.025T) \quad dBkm^{-1}kHz^{-1}$$
$$P_2=1-1.37x10^{-4}D+6.2x10^{-9}D^2$$
$$f_2=\frac{8.17x10^{(8-1990/\theta)}}{1+0.0018(\mathrm{s}-35)} \quad kHz \quad (11)$$

Pure water contribution

$$T \le 20^0 C$$
$$A_3 = 4.937x10^{-4} - 2.59x10^{-5}T + 9.11x10^{-7}T^2 - 1.5x10^{-8}T^3$$
$$T > 20^0 C$$
$$A_3 = 3.964x10^{-4} - 1.146x10^{-5}T + 1.45x10^{-7}T^2 - 6.5x10^{-8}T^3$$

$$P_3 = 1 - 3.83x10^{-5}D + 4.9x10^{-10}D^2 \tag{12}$$

4. The comparison of transmission losses between Normal mode and Francois-Garrison

As far as ocean waveguide is concerned, two layers are considered, i.e., water column and sea bed. The water column is bounded by sea surface and sea bed whereas the sea bed has different structures. It can be made of sand, mud or a composite of both of them.

In this paper, the parameters of Tonkin gulf are investigated, they are given in the table 1 as Follows

Table 1. The parameters of Tonkin gulf

Parameters	**Value**
Depth of water column	120 m
Depth of sea bed (made of sand)	10 m
Temperature (T)	10 degree
Salinity	35 %o
Theta	273+T
p.H	7.8

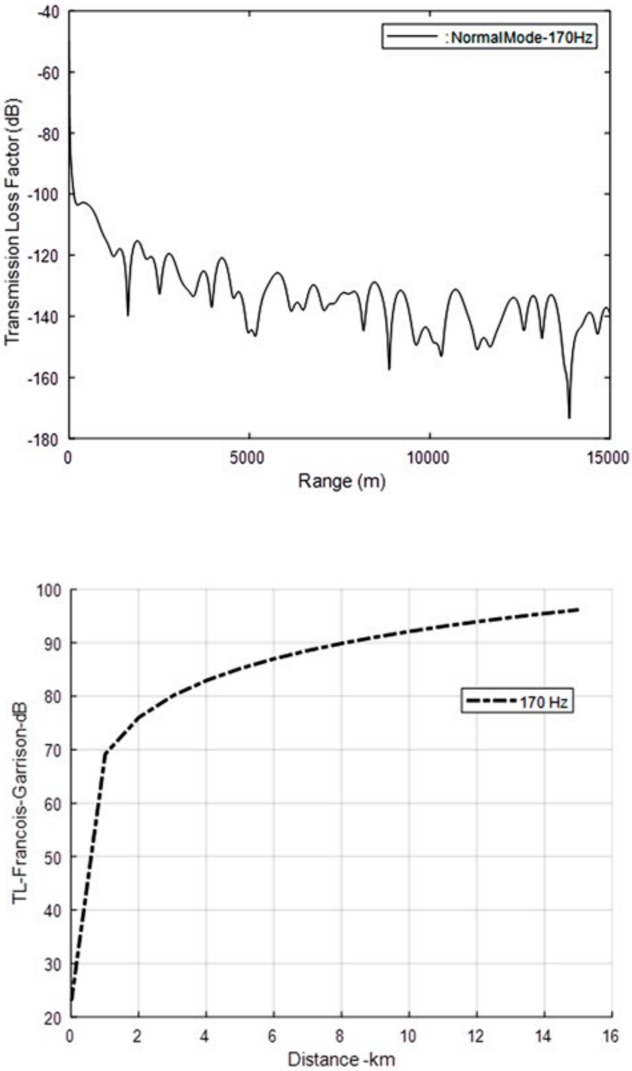
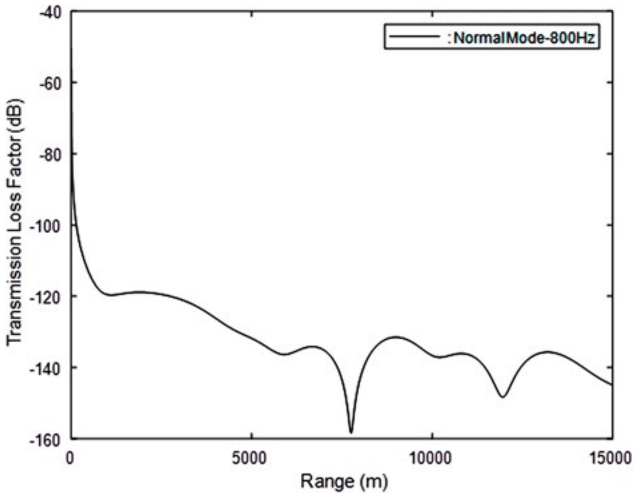


Fig 1. The Transmission losses of Normal mode and Francois-Garrison at 170 Hz



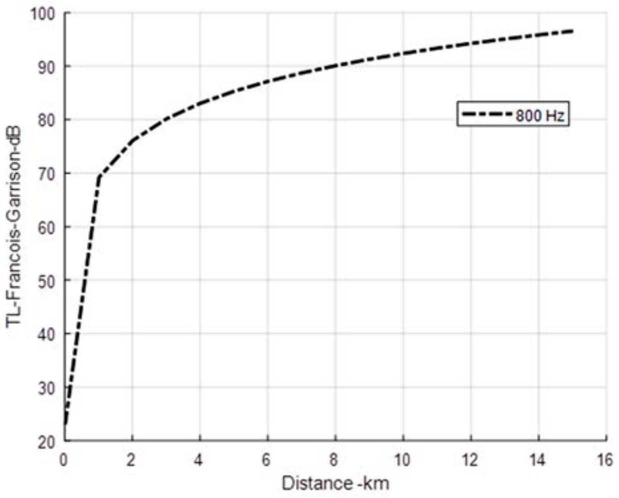


Fig 2. The Transmission losses of Normal mode and Francois-Garrison at 800 Hz

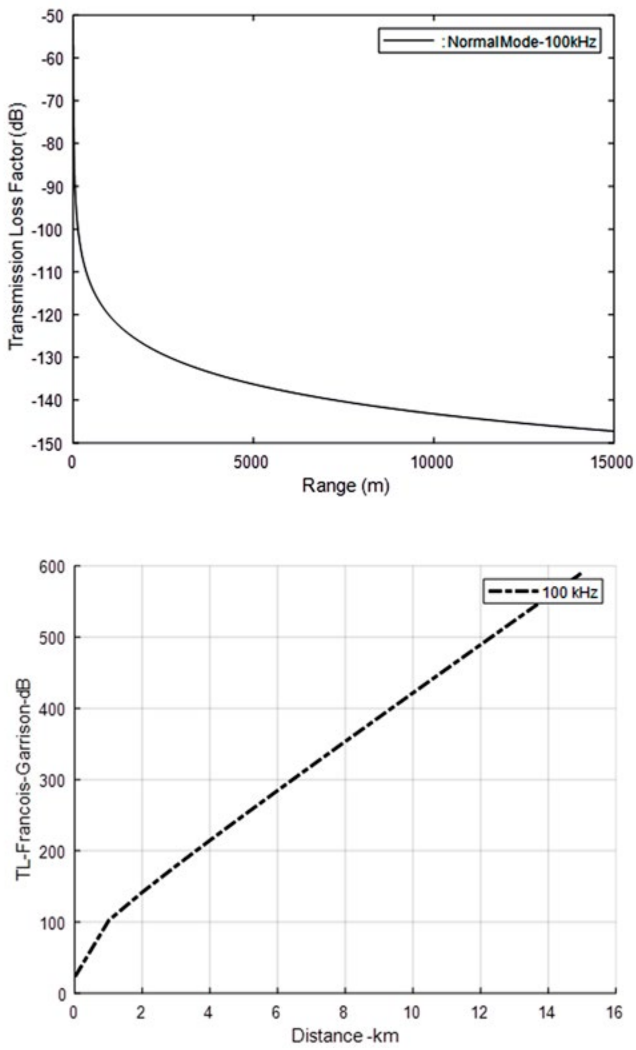


Fig 3. The Transmission losses of Normal mode and Francois-Garrison at 100 KHz

From Fig. 1 we can see that at at frequency 170 Hz <1000 Hz and at distance of 15 km the transmission losses of Normal mode is around 90 dB (50-140) whereas the transmission loss of Francois-Garrison is about 90 dB (25-95). So there is a consistent between these methods.

From Fig. 2 we can see that at at frequency 800 Hz<1000 Hz and at distance of 15 km the transmission losses of Normal mode is around 110 dB (30-140) whereas the transmission loss of Francois-Garrison is about 90 dB (25-95). So there is also a consistent between these methods.

At low frequencies we may conclude that for the optimum underwater transmission, the Normal mode is preferred.

From Fig. 3 we can see that at at frequency 100 KHz and at distance of 15 km the transmission losses of Normal mode is around 85 dB (60-145) whereas the transmission loss of Francois-Garrison is about 570 dB (20-590). There is a non-consistent between these methods. The reason for this non-consistent may be in Normal mode theory one considered that the water column is homogeneous for all acoustic vibrations, it is not true in reality.

At high frequencies, we may conclude that for the optimum underwater transmission, the Francois-Garrison should be used.

Conclusion

The SONAR applications relies much on the problem of underwater sound propagation. There is not only the theory of underwater sound propagation in ocean such as Normal mode but also the empirical model such as Francois-Garrison formula dealing with the problem. In this paper, we investigated the optimum sound propagation in ocean, the transmission losses of both Normal mode and Francois-Garrison are calculated and simulated at a wide range of frequencies (170 Hz, 800 Hz, 100 KHz). Under the simulation conditions, the results show that for optimum underwater sound propagation at low frequencies the Normal mode is preferred and at high frequencies the empirical formula of Francois-Garrison should be used. This finding is very important for the precisely calculation of underwater sound propagation

References

[1] Jensen, J. B., Kuperman, W. A., Porter, M. B., Schmidt, H.: Computational Ocaen Acoustics, Sringer Science, 2011

[2] Pekeris, C. L.: Theory of propagation of explosive sound in shallow water, Geol. Soc. Am. Mem. 27, 1948

[3] Ide, J. M., Post, R. F.,Fry, W.J.: The propagation of underwater sound at low frequenies as a function of the acoustic properties of the bottom, J. Acoust. Soc. Am, **19** (283)

[4] Newhall, A. E., Lin, Y. T.:A three dimensions underwater sound propagation model for offshore wind farm noise prediction, J. Acoust. Soc. Am. **5**(145), https://doi.org/10.1121/1.5099560, 2019

[5] Tran Cao Quyen, Hoang Viet Trung.: A 3D modeling of underwater sound propagation and its application in Tonkin gulf, J. Scien. Tech, **57**(2), ISSN: 1859-3585

[6] Francois, R. E., Garrison, G. R.: Sound absorption based on ocean measurements: Part I: Pure water and magnesium sulfate contributions, J. Acoust. Soc. Am, **72**(3), 896-907, 1982

[7] Francois, R. E., Garrison, G. R.: Sound absorption based on ocean measurements: Part II: Boric acid contribution and equation for total absorption, J. Acoust. Soc. Am, **72**(6), 1879-1890, 1982

[8] Schulkin, M.: Eddy viscosity as a possible acoustic absorption mechanism in the ocean, J. Acoust. Soc. Am, 35, 253, 1963

[9] Schulkin, M., Marsh, H. W.: Sound absorption in sea water, J. Acoust. Soc. Am, 34, 864-865, 1962

[10] Thorp, W. H .: Deep-ocean sound attenuation in the sub and low kilocycle-per-second range, JASA, 38, 648-654, 1965

Tran Cao Quyen (Ph.D)

is the member of Faculty of Electronics and Telecommunications, University of Engineering and Technology, Vietnam National University, Hanoi (VNUH), Vietnam. His major subject consists of Antenna and propagation, RADAR, Underwater Acoustic, and SONAR. Dr. Quyen is the author over 30 technical papers and supervises 2 Ph.D students, 3 master students and dozens of Bachelor in Electronics and Telecommunications.

Over his professional career, he successfully combined academic teaching with practical application of the results of his research. For instance, his product, namely, **OFDM underwater acoustic modem using IC technology** has been received much attention from Vietnam National University as well as Vietnam Navy.

For the problem of underwater source localization, he interested in **Riemannian Matched Field Processing** and suggested to Czech Republic Navy as well as Vietnam Navy using a passive SONAR system which is embedded the proposed algorithms.